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A Grammatical Note on Utzon's Vaults

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Abstract
This paper outlines an analysis of Utzon's process of ideating vault through the lens of shape grammar, from investigating the eureka moment of the spherical schema for the Sydney Opera House to interpreting vault schema generation in other, later projects. The results show that most of Utzon's vault's schemas can be generated with only a few rule schemas, and that the generated schema can produce various parts in the designs. This interpretative study can help to understand not only how an architect develops schemas to solve various problems within a project but also how principles from an earlier schema can be reused to formulate vault schema for different contexts.

Keywords Jørn Utzon · Shape grammar · Vault · Sydney opera house · Farum town center · Bagsvaerd church · Kuwait national assembly

Introduction

The success of Jørn Utzon's spherical schema for Sydney Opera House (SOH) seems to eclipse the fact that it took a very long time to formulate the schema. Utzon's spherical solution was proposed four years after he won the SOH competition. The project suffered significant construction delays and a ballooning budget as a result of the lack of clarity in defining the geometric principles of the construction. According to the jury, "the drawings submitted for the [competition] scheme were simple to the point of being diagrammatic" and for the engineer, "apparently unaided by structural engineering advice…all surfaces were free shapes without geometric definition" (Arup and Zunz 1973, pp. 4–5). From the engineer's perspective, Utzon's initial idea of using a thin shell for the roof "was not possible since the very shape of the roof introduced high bending moments regardless of any structural system" (Arup and Zunz 1973: 5). After years of iterations, from a series

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of parabolic to ellipsoid schemes, and from a thin shell to rib vaults, the spherical schema was finally formulated in 1961 and was proven to efficiently rationalize the design for the construction process (Mikami 2001). Nevertheless, given its canonical reputation, morphological studies on how the spherical schema evolved are rare.

This paper aims to understand how the geometric principles of vault design evolved into an explicit schema. We maintain that understanding the ideation process that lead to the final schema is important in learning how to formulate a broader range of geometric principles for vault design. The geometric clarity of Utzon's post-SOH projects suggests that his success in producing effective schemas increased; this can be seen in his designs for the Bagsværd Church (BC) and the Kuwait National Assembly (KNA) (Utzon et al. 2005; Utzon 2008). Although the scale of these projects was modest and less complex than that of the SOH, the time taken to prepare their schemas was much shorter.

Method

This study uses literature, primarily from Yuzo Mikami's *Utzon Sphere* (2001) which contains extensive and comprehensive diagrams that show how the spherical schema works; Ove Arup and Jack Zunz's report in *The Arup Journal* (1973); and Richard Weston's *Utzon: Inspiration, Vision, Architecture* (2002), among other sources. We also look at *Jørn Utzon Logbooks* particularly the ones that documented BC (Vol. 2) and KNA (Vol. 4) in detail, as well as drawings from the Utzon Archive (Utzon et al. 2005; Utzon 2008).

To capture Utzon's ideation process we used shape grammar analysis, a computational design method to interpret design transformation process into a set of shape rules, such as rule M: □ → ◇ or rule N: □ → ◫. Comparable principles used in different rules can be encapsulated in a rule schema, for instance, $x \rightarrow \mathrm{t}(x)$ or $x \rightarrow \mathrm{div}(x)$, where x represents a shape, $\mathrm{t}(x)$ represent a transformed shape x, and $\mathrm{div}(x)$ represent a divided shape x (Stiny 2011). Rule iteration can be reordered to produce different designs (e.g., M-M-N-N or M-N-M-N). A design corpus can be "locked" into a parametric schema to constrains a design within certain parameters and conditions (e.g., spatial proportion, structural constraints, and environmental conditions). Here, we used shape rules with different iterations to evaluate their capacity to produce a variety of Utzon vault typologies, whereas a parametric schema was used to capture a certain proportion system in his vault designs.

Cross-Dimensional Embedding

Utzon's ability to embed shapes across dimensions seems to have significantly contributed to the conceptualization of his schema. Shape grammar can assist in understanding this embedding process and in interpreting important aspects of Utzon's formalization process from his intuitive designs. It categorizes the shape geometry (U_{ij}) based on the shape's dimension (i) and the spatial dimension where the shape exists (j) (Stiny 1993); for example, the shape in U_{12} indicates the existence of a one-dimensional shape in a two-dimensional space. The embedding process of the shape in U_{ij} can occur within the same dimension ($i=j$) or different dimensions ($i+1$ or $i-1$). For instance, seeing a part of one-dimensional line, abbreviated with prt(x), indicating $i=j$ because the part shares the same dimension as the line in U_{11}. Conversely, cross-dimensional embedding involves seeing the boundary of a shape x, annotated with b(x). For example, seeing a square from four lines, as the square's boundary, means seeing a shape where $i=i+1$ as the square is obtained in a higher dimension (U_2). In contrast, embedding a line from a square's edge indicates that $i=i-1$ (Stiny 2011). Embedding towards a higher-dimensional shape or inversing a shape's boundary, $b^{-1}(x)$, is more challenging because many higher-dimensional shapes can be bounded by a shape; for example, a line can be perceived as a boundary of a square, a triangle, or any other polygon. This might be one of the reasons that it took so long to arrive at a spherical schema for the SOH, as the roof curves in the original design could be embedded with elliptical, paraboloid, or spherical surfaces in a higher dimension.

This paper outlines our investigation of the process of vault ideation in four parts. The first part revisits Utzon's ideation process when formulating the SOH schema for the main vaults and the Minor Hall ceiling. The second and third parts investigate schemas development for BC and KNA, to see how the underlying principles in the SOH schemas might have evolved in different contexts. The fourth part investigates possible schemas for his unbuilt design, Farum Town Centre (FTC), as an exercise to apply findings from the first three parts.

Sydney Opera House Vault Grammar

In shape grammar terms, the SOH spherical schema is a parametric shape which produced vault designs by subdividing the sphere's surface using its great and small circles (Fig. 1). While the resulting spherical schema is explicit and self-explanatory, Mikami's notes on Utzon's ideation process are nonvisual; his textual narrative is based on a conversation he had with Utzon a few months after the invention of the spherical schema. According to Mikami, the sphere idea struck Utzon when he was alone at his model shop in Hellebaek, Denmark, in the summer of 1961. Utzon "stacked them [the shell models] together one by one, a smaller shell inside a larger one. When he finished…the curvatures of the shells were much more similar to each other than he had thought all these years" (Mikami 2001, p. 65). This surface-level geometric resemblance led Utzon to presume that "if they were so similar, why couldn't they be cut from one shell to the other? In order to do that the curvature must be the same in all directions. What is a geometrical body with a constant curvature in all directions? A sphere!" (Mikami 2001, p. 65).

Utzon's ideation process is outlined in Fig. 2 using a shape rule format with the state of the shape's dimension in U_{ij}, to show how the shape rose to a higher dimension from a set of curves embedded on the stacked vault's model (U_{12}). The growing curve in step 2 indicates the use of an embedding process to create a larger shape; the shape forms into a circle in step 3, as the curve's maximum shape. The defining moment occurs in step 4, where the curve no longer occupies

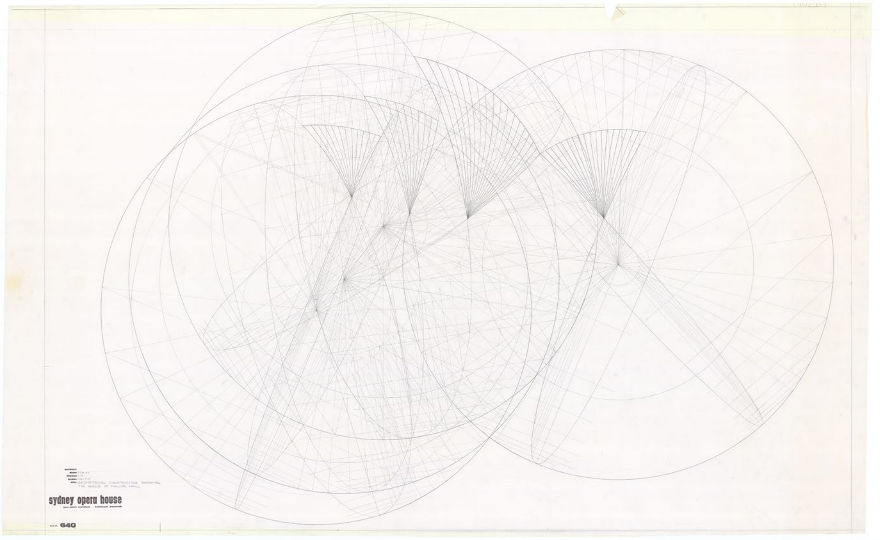

Fig. 1 Utzon's spherical schema, drawn by Rafael Moneo (Utzon 1962b) © Utzon Archives/Aalborg University and Utzon Center

a two-dimensional space but rather exists within a three-dimensional sphere (U_{13}). The last two steps were rewritten into a shape rule to express Utzon's findings as follows: given a curve, create a sphere where the curve lies on the sphere's surface as its boundary.

Utzon furthered his thought process by "rush[ing] home and [taking] a child's rubber beach ball" (Mikami 2001, p. 65). This is a crucial remark because a typical beach ball design is already subdivided into several spherical triangles with two circles at each pole painted with alternating colors (e.g., white and red). Figure 3 shows an iteration where a beach ball serves as the initial shape; it is followed by the embedding process to retrieve the cutting planes (U_{23}) from the ball's great circles (U_{13}), marked by the border between the different colors. The minimum number of cutting planes required to form the vault (i.e., two) is captured in step 3, and the spatial relationship between the plane and the sphere is stated in steps 4 and 5. The last two steps are formalized into another shape rule, where the beach ball's great circle (GC) split a sphere with a plane.

The third side of the vault was defined when Utzon "put [the beach ball] into a bathtub full of water" (Mikami 2001, p. 65). Step 2 in Fig. 4 shows the water's surface (U_{23}) as the third cutting plane, dividing the dry and wet areas on the ball to produce the small circle boundary (U_{13}) as part of the vault's third side. As the ball is submerged deeper, it brings the water's surface closer to the ball's centroid and increases the radius of the small circle. Step 3 shows a part of the ball that established the SOH vault divided by two great circles and a small circle. The last two steps were formalized into a shape rule, parameterized by the distance between the ball's centroid and the small circle's centroid to modify the small circle's size and

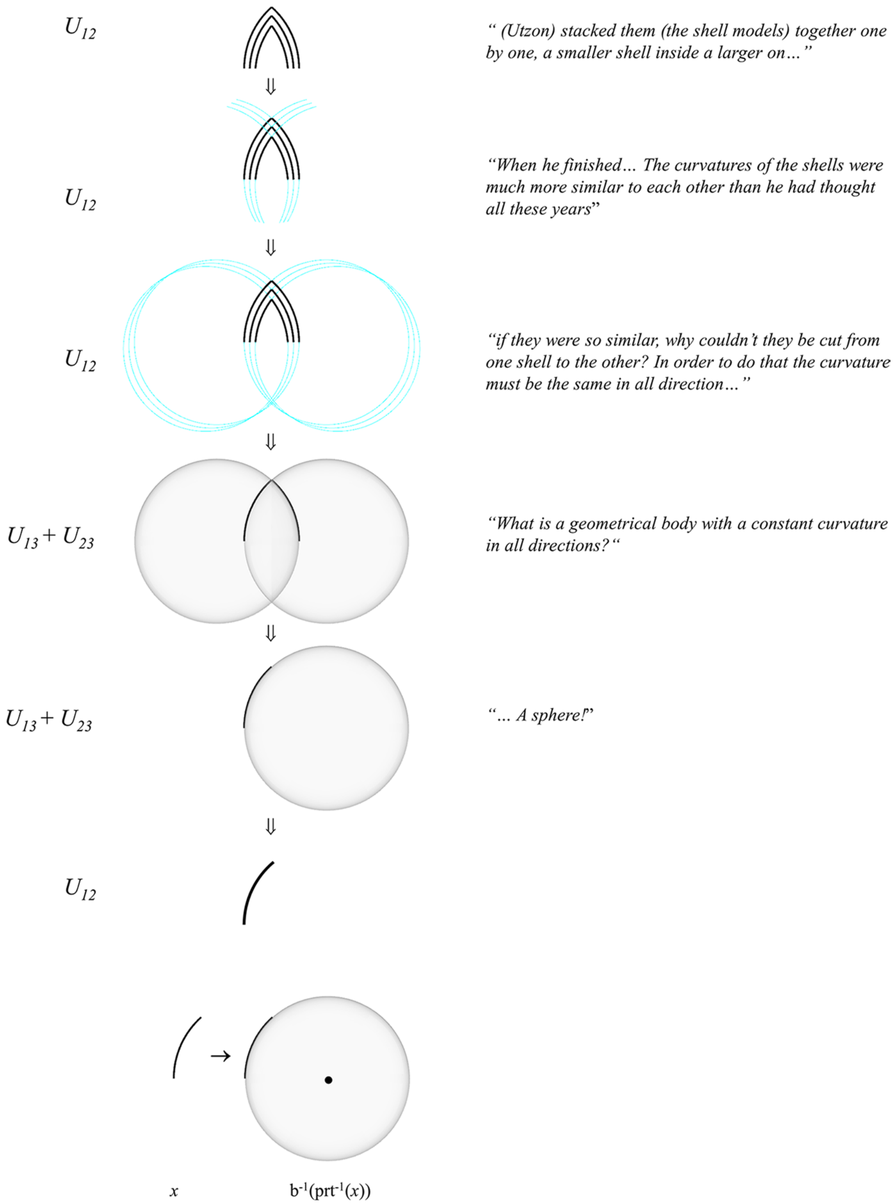

Fig. 2 Adaptive Ruleplay. An interpretation of Utzon's ideation moment from the stacked model at the workshop and the formalized rule at the bottom. The rule returns an inverse-part of a curve (prt(x)), i.e., a circle, and then return an inverse-boundary of a circle, $b^{-1}(\text{prt}(x))$, i.e., a sphere

position: the larger the distance, the smaller the circle and spherical triangle area. The findings at this step complete Utzon's quest to formulate the SOH vault schema; by then, Utzon was "able to see the shapes of spherical triangles he could cut out

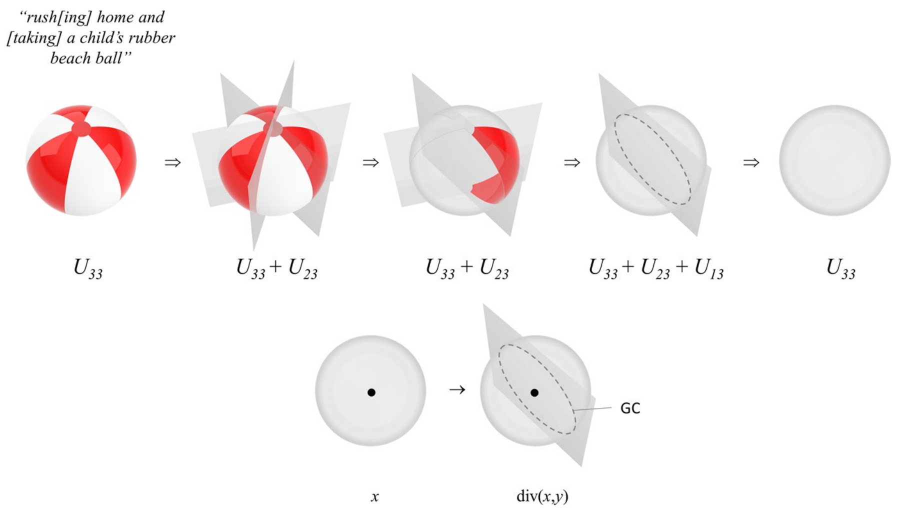

Fig. 3 Interpreting the embedding process from a beach ball design into a subdivision rule that produces a plane to slice the ball's and return its great circle (GC). The way shape x is divided by shape y is annotated with a rule schema $x \rightarrow \text{div}(x, y)$

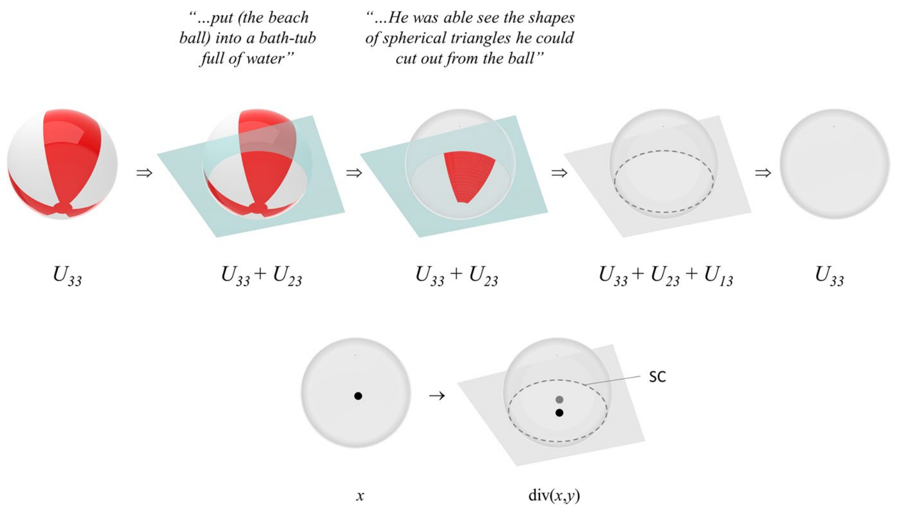

Fig. 4 Illustration from Utzon's bathtub experiment with a submerged beach ball seen from below and the subdivision rule that return the ball's small circle (SC). The two points indicate the distance between SC and the ball's centroid

from the ball … [Utzon] realized that the variety of shapes and sizes available was almost limitless, big and small, flat and upright" (Mikami 2001, p. 65).

Our interpretative study produced a set of rules for the SOH grammar: R1 creates a sphere, capturing Utzon's eureka moment in the model workshop; R2 slices a sphere with its great circle to interpret what he saw in the beach ball; and R3 slices a sphere with its small circle to recreate his bathtub experiment. Additionally, we added R4 to copy and R5 to move the sphere into a new position (Fig. 5). Rule D1:  →  is also added to capture the spherical triangle embedded in a sphere, i.e., "the sail". To increase their versatility, the rules are accompanied with rule schemas to represent their underlying principles. R1 is described with a schema $x \rightarrow \mathrm{b}^{-1}(\mathrm{prt}^{-1}(x))$ to return a three-dimensional shape of a curve by revolving a shape around the inverse part of the curve, i.e., a circle. For example, revolving a line will produce either a cylinder or a cone (depending on the line's orientation angle). Revolving an arc could produce a pointed dome and revolving a circle will produce either a sphere or a torus (based on the circle's size and whether shape x and y share a center point or not). As such, a circle's inverse boundary in U_{23} or U_{33} is open for interpretation; it could be a sphere, cylinder, cone, torus, dome, etc. By assigning a different shape into the rule schemas, various rules can be defined to analyze other of Utzon's designs. R2 and R3 are described with a subdivision schema $x \rightarrow \mathrm{div}(x,y)$ to subdivide shape x with shape y. R4 is described with schema $x \rightarrow x + \mathrm{t}(x)$ to copy and transform a shape and R5 with $x \rightarrow \mathrm{t}(x)$ to transform a shape with translation.

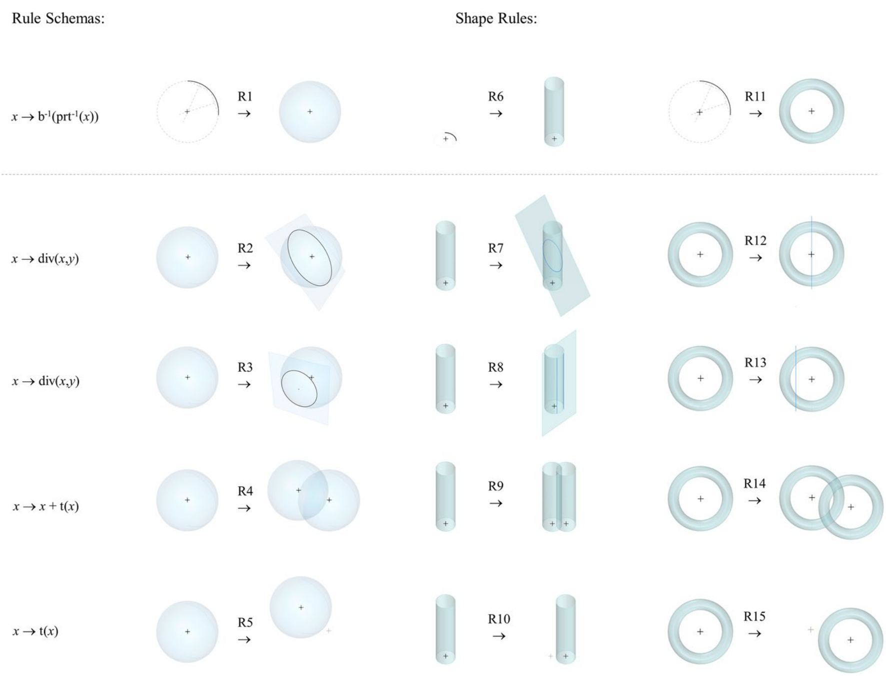

Fig. 5 Shape rules for SOH spherical schema (R1-R5) and other possible rules (R6-R15) derived from the same rule schemas in the left column. Different three-dimensional shapes that have a circle as its boundary (top) were assigned into the rule schemas as shape x to generate different rules

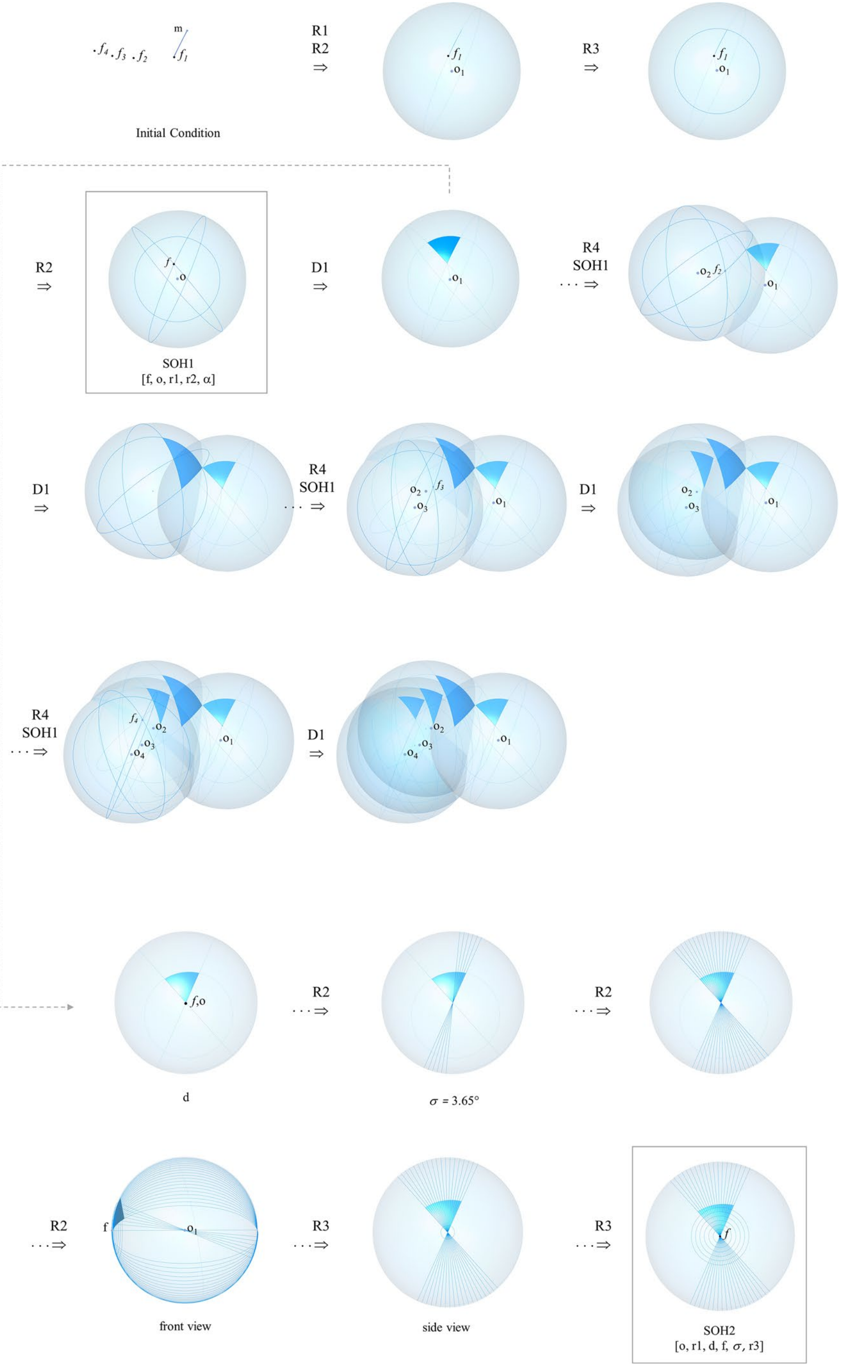

Fig. 6 Rewriting the spherical schema (SOH1) and the ribs schema (SOH2). The initial curve f_1-m (top left) rest on the existing foundations (f_{1-4}), built before the spherical schema was invented. Spherical schema application in SOH can be regarded as parametric drafting to match the modified design with the existing structures

Sail Schema

To demonstrate the SOH grammar, the shape rule and parametric schema work in tandem to regenerate the SOH design. As the rule generates new designs iteratively, at any given step a generated design can be "frozen" into a parametric schema and constrained by parameters as a template to replicate similar designs with certain proportions. As can be seen in Fig. 6, the iteration R1–R2–R3–R2 initially produced a spherical triangle by slicing the sphere with its great and small circles, which was then frozen into a spherical schema (SOH1) with parameters *f* (foundation position in the podium), *o* (the sphere's centroid), *r1* (the sphere's radius), α (great circle angles) and *r2* (small circle distance to the sphere's center). By creating new spheres with R4 in the next steps, SOH1 can be applied to each of these spheres repeatedly to generate the other SOH sails (Fig. 6). D1 might seems trivial in this iteration. However, given eight spherical segments in SOH1, iteration of D1 can lead to many different designs, such as $\frac{8!}{(2!(8-2)!)}$ or 28 possible combinations of two-segments, 56 combinations of three segments and 70 combinations of four segments (Fig. 7).

Ribs Schema

If we continue the iteration after the SOH1 formulation along a different path, by repeating R2 and R3 several times on the spherical triangle, we obtain a new schema (SOH2). Here, the R2 iteration split the spherical triangle into radiating ribs with equal rotation angle interval at 3.65°, pivoting at *o-f* axis. R2 then continue to split each rib into panels by pivoting at axis perpendicular to *o-f*, creating a 6.5 m height for the sail's pedestal, followed by standardized rib modules 4.6 m long, and topped by a ridge beam segment (Arup and Zunz 1973; Mikami 2001). The segmented ribs provide radius values for R3 to split the other ribs in parallel with small circles. SOH2 works in a way that is similar to the Earth's longitude and latitude grid. It is constrained by parameters *o, f,* and *r1* from SOH1 with input *d* (a spherical triangle), σ (great circle angles), and *r3* (small circle radius). If intervals of σ and *r3* are constant, SOH2 can slices the spherical triangle into a set of uniform panels that share the same angle and radius, and can, therefore, be prefabricated with only a few precast molds, as in the case of SOH.

Tiling Schema

According to Utzon, it was the tiling problem in a full-scale mock-up of the SOH's earlier scheme that encouraged him to pursue a precise and economical geometric solution (Utzon 1965). One that led to a 120×120 mm^2 square tile-module, arranged diagonally to clad the roofs. To interpret the process of tile patterning, the tile is rotated 45° on the rib's axis and fitted at the bottom near *f*, so that its vertices touch the rib's edge (with known σ and tile's length (*l*), the distance between the tile's bottom vertices (*a*) and *f* can be obtained with trigonometric ratio). R2 then generates a tile grid based on the tile's edge orientation (Fig. 8).

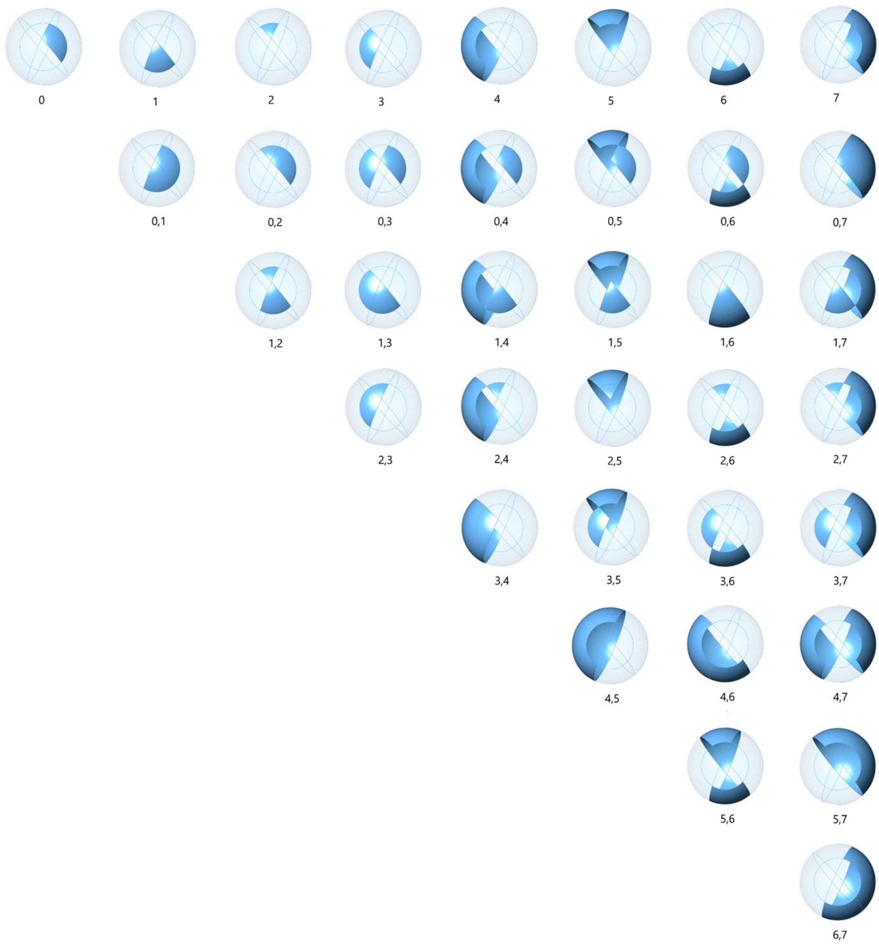

Fig. 7 Example of D1 iteration results from schema SOH1: eight segments at the top (0–7) and 28 combinations of two spherical triangles

We interpret the tile coloring rule based on whether the rib's edge trims a tile or not: if a tile is not a square, then they are colored in blue; otherwise they are white. This rule generates odd-number patterns driven by the number of white tiles along the grid axis, i.e., one-tile, three-tiles, five-tiles, seven-tiles, and so on. Based on this interval, we add more coloring rules such that if the number of white tiles in the next row is larger, then they are colored in blue. The approximate result in Fig. 8 shows the emergent chevron-like panels produced by the coloring rules. As it was built, the non-square shapes along the ribs' edges were merged into one tile. This generated 41 types of tiles, with a total of 1,056,006 tiles grouped into 4228 precast tile panels (Mikami 2001; The Sydney Opera House Trust 2020). Most of these panels are identical and can be cast repeatedly using only 26 chevron molds (Arup and Zunz 1973).

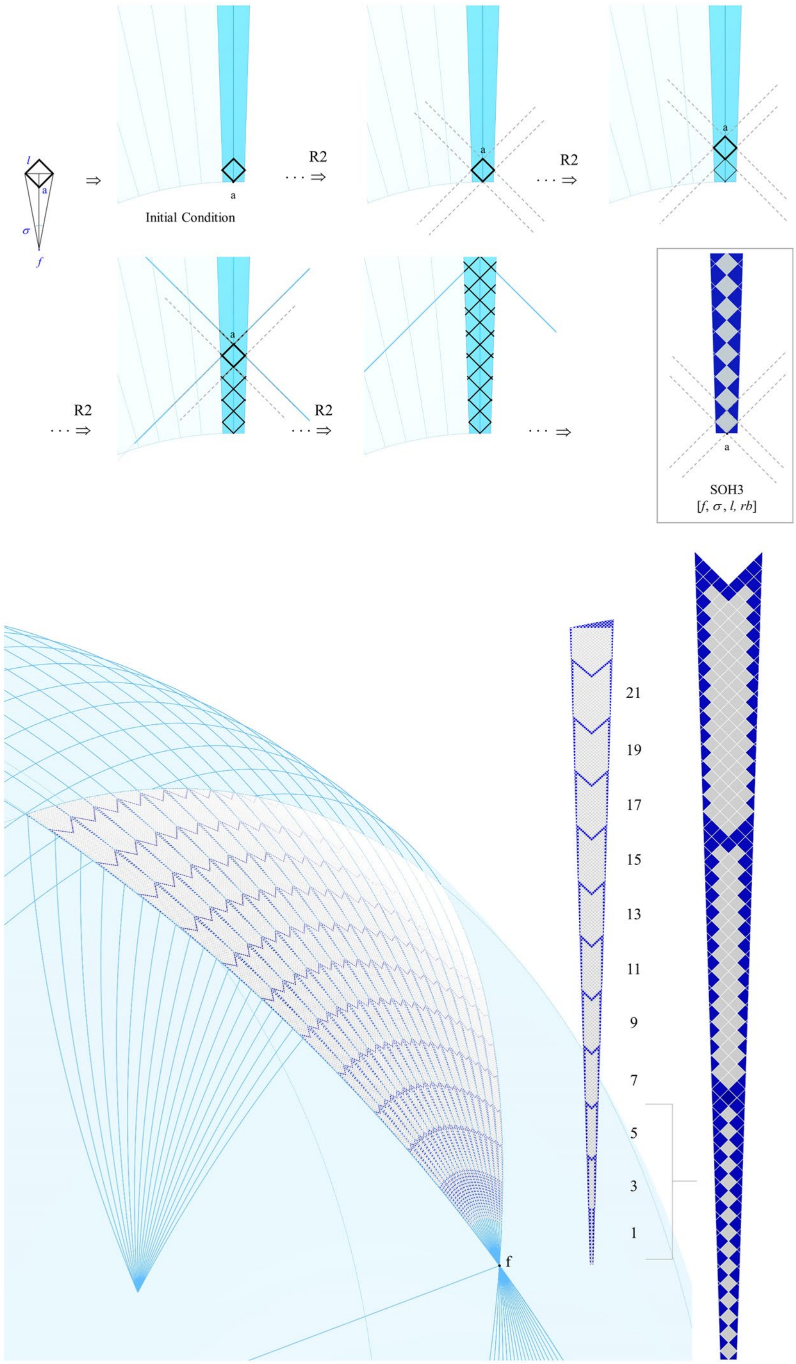

Fig. 8 The Utzon tiling overlays the subdivided ribs from schema SOH2. The tiles diagonal grid is formed by the sphere's great circles, starting at point *a* in the top left (obtained using trigonometric ratio), and propagates along the rib's axial line. SOH3 summarizes the tiling grid design schema. The chevron shapes emerging from the colored grid indicates the subdivision for the precast tiles lid panels

Minor Hall Ceiling Schema

As shown in *The Yellow Book* (Utzon 1962a), Utzon's early design for the Minor Hall acoustic ceiling featured large convex and concave curvatures resembling an ocean wave (Utzon 1962a). Later, based on Lothar Cremer and Werner Gabler's advice to avoid an undesired effect of the sound being too focused, the design was replaced by smaller convex surfaces formed by a series of cylinders (Weston 2002). For Utzon, the way a sound travels is analogous to the sunlight glimmering on the water as the light is bounced to our eyes by the moving water (Leplastrier 2017).

Figure 9 illustrates the generation of the Minor Hall schema in response to sound location at a stage. With the seating contour and point *s* near the stage as an initial condition, R6 creates a cylinder that coincides with *s*. R9 then creates another cylinder centered at *p* in step 2, as a guiding shape that constrains the first cylinder. In step 3, R9 produces another cylinder that intersects with the first one to define a ceiling segment: arc *s-p*. Based on this iteration, we define rule R16 with label *s* and *p* by combining the results from step 1 and step 3, where *s* moves to a new position to allow R16 generate more ceiling segments recursively. The spatial relationship between the guiding shape and the cylinder is formalized into rule R17.

The Minor Hall design was divided into ten sections, arrayed radially in floor plan with 5° intervals. Figure 10, showing the R16 iteration for a middle section, makes it clear how the ceiling generation affects the direct sound radiating from the center stage. As seen in the iteration, the radiating line starts to bounce off the ceiling on

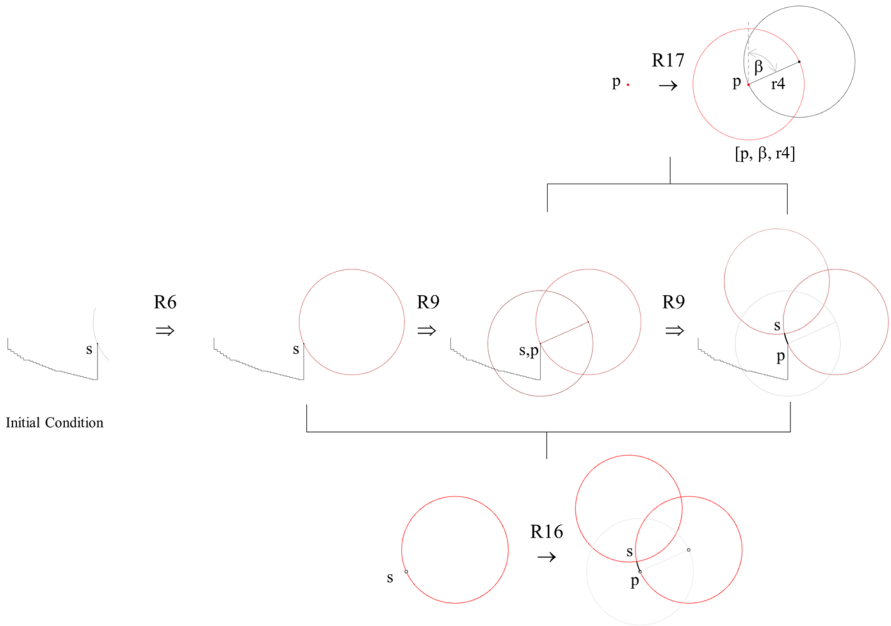

Fig. 9 Minor Hall acoustic ceiling rule generation, starting with a seating contour in section view and indicative ceiling curve at point s (left). The circles represent a cylinder side view to generate the ceiling panel

the third step and then continues from the fifth step onward until it reaches the back row. Each step in the R16 iteration marks p_n position, as an input for R17 to generate more cylinders for the adjacent sections (variable β lists a series of angles to locate those cylinders) (Fig. 11). As the Minor Hall ceiling was generated using cylinders with the same radius, they can be prefabricated with the same template.

The way in which the rules can be used to generate different parametric shapes (e.g., for modifying the acoustic ceiling, varying the roof shapes and generate the tiling grid) implies that the SOH spherical schema is just one of many schemas that can be interpreted from Utzon's ideation process. His ability to produce a variety of parametric schemas for different purposes was the key to the SOH's completion and constructability, and it is worth investigating further how this ability might have been applied to other projects. In the next sections, we revisit the vault design in his three other projects: Bagsværd Church (BC), Kuwait National Assembly (KNA) and Farum Town Center (FTC).

Bagsværd Church Schema

Echoing principles similar to the SOH Minor Hall, the vault in the Bagsværd Church (BC) is inspired by Hawaiian cloud (Utzon et al. 2005). Yet, unlike Minor Hall's wooden ceiling, which was intended to be hung from the roof, that of the BC vault is made of 8–10 cm thin concrete shells that span 17.35 m, without central columns, and rest on two bearing walls at end-span (Jensen 2005). The undulating shells work as a series of barrel-vaults to support the outer roofs and to diffuse the sound and light inside. While the concrete shell was cast in situ, economic considerations are apparent. The extrusion of the vault's profile into a single-curvature shell helps to standardize the wooden formwork and the prefabricated steel bars. An additional roof truss and acoustic reflector are not required, as the shell both supports the outer roof and reflects sound. Moreover, unlike the fully enclosed Minor Hall, BC vault welcomes the Scandinavian daylight. According to Mortensen (2005), the vault curvature caters to different types of sound. For example, the vault above the organ is higher to provide a longer-lasting reverberation and has a concave shape that is tilted away from the floor to minimize undesirable focusing of sounds, whereas the vaults above the congregation are convex and lowered to allow reflected natural speech to reach the back rows, balanced by sound-absorptive materials, e.g., carpets and textiles.

To interpret the ideation of the BC vault, we traced its evolution from its earlier scheme (Fig. 12). Utzon presented his *geometrisk bestemmelse af lofter* (geometric determination of ceilings) for the BC in May 1969, "formed on the basis of some drawings of circles … of varying cylindrical sizes and curl up at the ends, where you can get into the body of the church" (Utzon et al. 2005, p. 116). However, we found that the design transformation between 1967 and 1968 shows a more profound reasoning that explains the final design schema. Initially, the vaults were narrowed to the top, based on a calligraphic form, and their tip was visible from the street. There was no outer roof and more skylights enter the building through the gaps between the shells, where the circle of the shells'

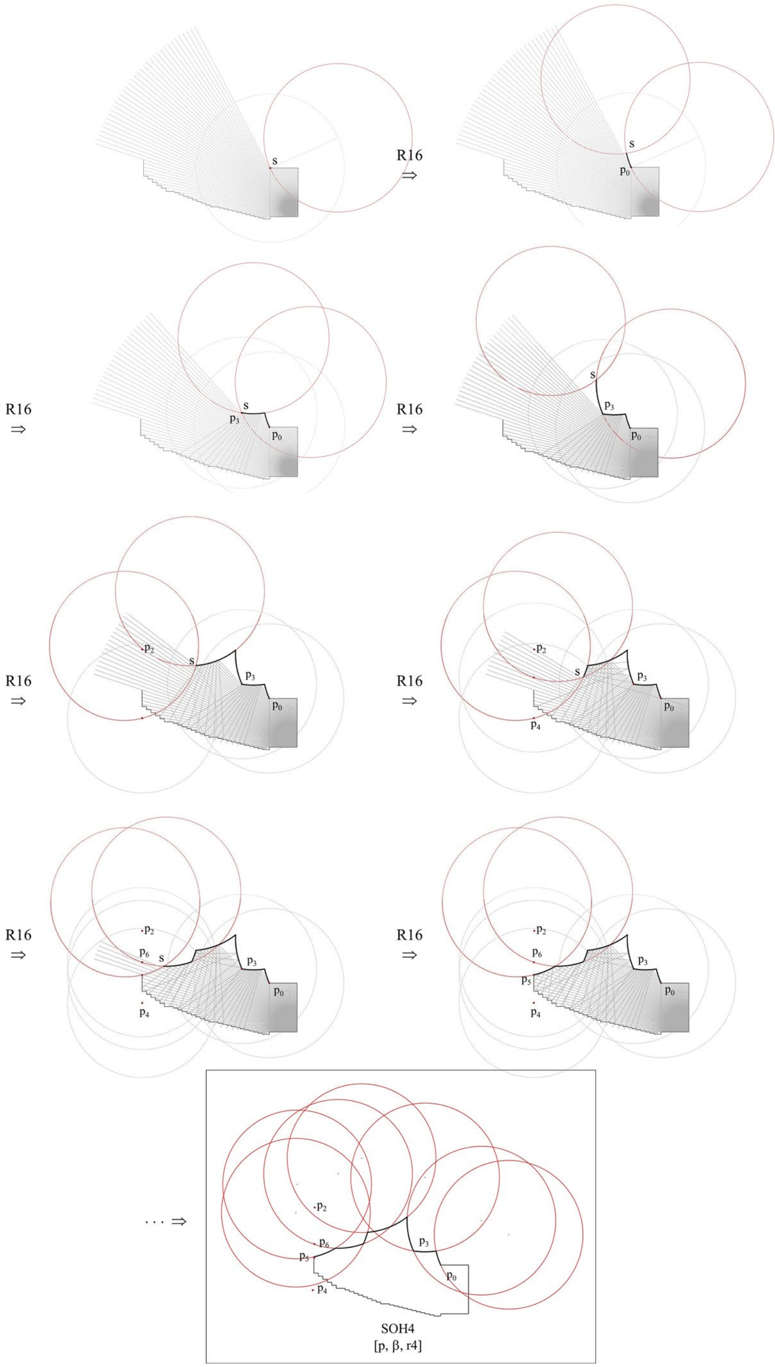

Fig. 10 Wave forming. Minor Hall ceiling iteration with R16, from the stage (top left) to the back rows (bottom right), summarized into SOH4 schema (bottom). The radiating lines mark direct sounds coming from the stage before they reflected by the generated ceiling in the next steps. Point p_n from this iteration becomes the input to generate the adjacent ceilings in the next figure. Adapted from SOH850 drawing (Utzon 1962a, b)

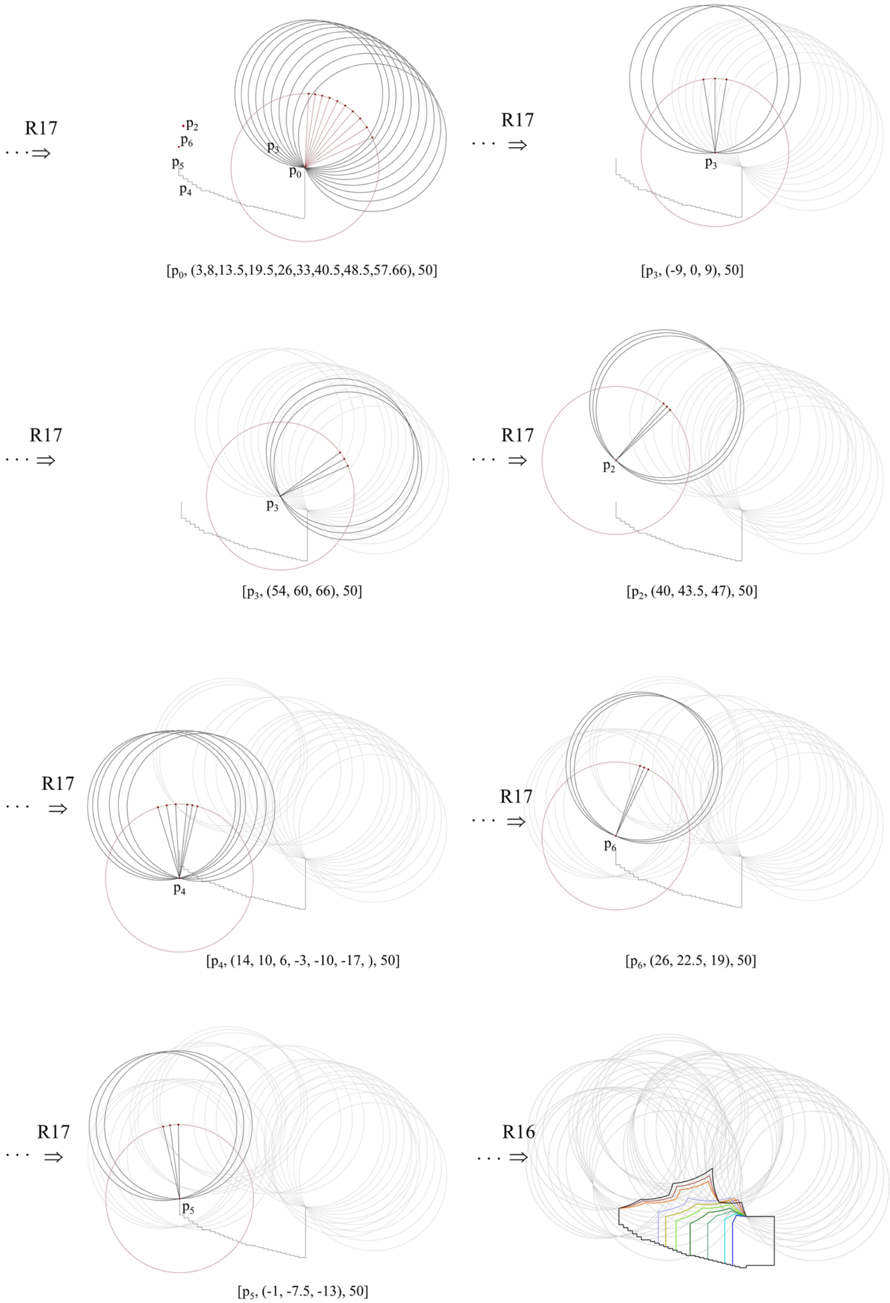

Fig. 11 Minor Hall acoustic ceiling iteration with various angle to generate the cylinder for the other sections. An example result is approximated on the bottom right. Adapted from SOH850 and SOH851 drawings (Utzon 1962a, b)

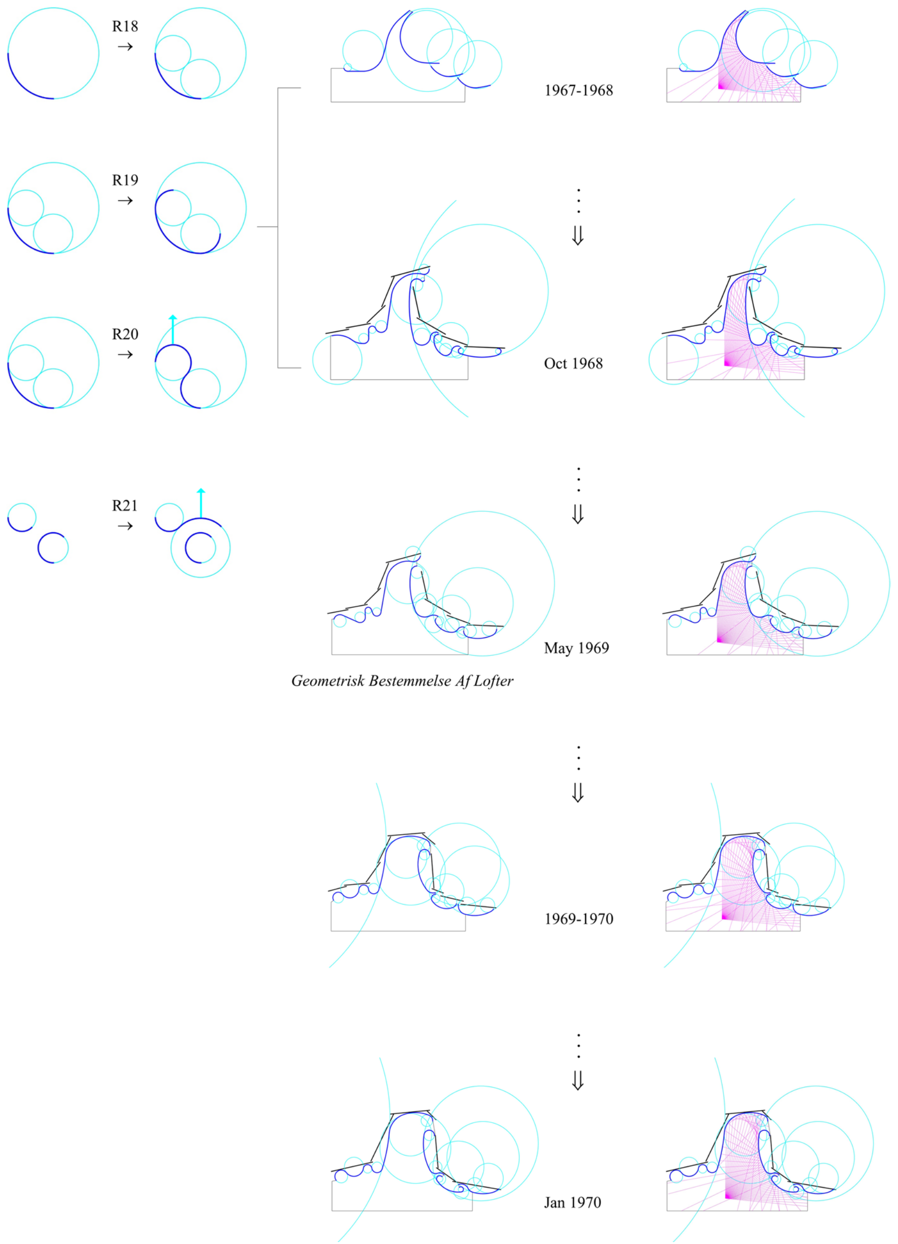

Fig. 12 The evolution of the BC vault based on drawings from Utzon Archive (Utzon, 1967–1970). The sound rays on the right (magenta) indicate attempts to maintain the acoustic quality for each version. On the upper left, rule R18 to R21 interprets the vault transformation from early design to the October 1968 version, where the vault's upper part supports the roof and its lower part reflects the sound inside

curvature seemed more apparent. Along with Utzon's sketches portraying a crowd below the clouds, the schema of the BC vault was essentially defined in October 1968, when the vault was presented with undulating surfaces and a covering of flat roofs. The design's transformation from then to the final design appears to be a parametric optimization, where the radii and positions of the cylinders were varied to optimize light and sound. This transformation has been interpreted into rules R18 to R21 (Fig. 12). R18 inserts two cylinders into a cylinder. R19 rolls a cylindrical segment inward and maintains its convex curvature. R20 undulates a segment into a concave-convex surface. R21 extends the rolled segment by following the curvature of the adjacent edge to provide support for the weathering roof.

Figure 13 shows an example of the iteration of these rules. Starting from Utzon's early sketch, showing two big clouds, R6 returns six cylinders to represent the clouds' curvature, followed by a parametric adjustment of cylinders *i*, *ii*, and *iii*. R18 then adds small cylinders, R19 rolls the cylinders' segment above the congregation, and R20 warps the segment above the altar and balcony. The BC schema is summarized in step 4. Finally, R21 is applied to extend and warp a segment to support the roofs. This results in a series of undulating vaults, optimized for structure, acoustics, and daylight.

BC's cylindrical schema adapted its respected context, material, and function. The chosen construction method, cast-in-situ concrete, could have led Utzon to treat the BC vault as a continuous and pliable parametric shape to achieve the ephemeral cloud-like shell, unlike that of Minor Hall, which was divided into segments to achieve an efficient prefabrication method.

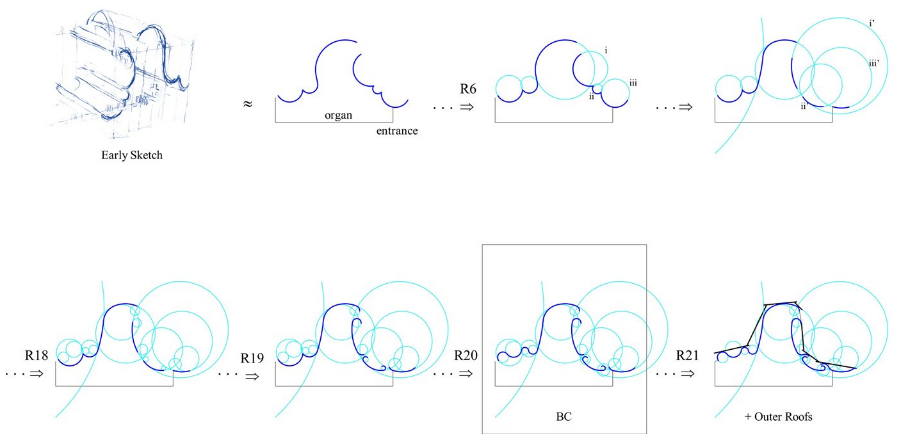

Fig. 13 Interpretation of the development of the BC schema based on Utzon's sketch in 1968 (Utzon et al. 2005)

Kuwait National Assembly Schema

The objective of integrating context and construction continued on the KNA project. This section discusses a developmental process for the cylindrical schema of the components of KNA's main street, assembly hall, and public square. As a result of Utzon's collaboration with structural engineer Max Walt, the cylinders in KNA appear in the design development phase. This departs from a winning scheme that was dominated by flat Arabic arches. Utzon cited several references for KNA's cylindrical structures, namely Mallorca's windmill, bamboo, and Yingzhao Fashi (Utzon 2008). Yet, an important note from Johan Fogh, a former member of Utzon's staff, indicates that, in 1973, Utzon initially had an idea to use a "hollow column". This was shortly followed by a modeling exercise with paper, bottles, and cardboard tubes that were trimmed at the top like a bishop's miter and later cut in half (Fogh 2008, p. 83). This exercise suggests an intention to optimize the structure and construction method from the outset. The trimmed cylinders would reduce unnecessary weight and material based on vertical load and by doing so, with the same diameter for the other components, it would also reduce the number of unique parts for prefabrication.

This early exercise has been interpreted with the rule schemas used in the SOH (Fig. 14). Firstly, R6 returns a cylinder from a circle. R7 then slices the cylinder into segments, from which the columns in the main street and in the assembly hall and public square can be obtained (rectangular columns were added to transfer the vertical load efficiently and to minimize deflection). If the iteration continues with R9 by adding more cylinders in a horizontal orientation, we get more segments to obtain the main street beams. The iteration shows that, in theory, most of the KNA components can be generated from the same cylinder and adjusted using certain parameters (e.g., by adding more parts to increase length and height, or by cutting parts at different angles to optimize the structure). Understandably, more adjustment is required in practice to suit different functions and load distributions and further cost justification is required to decide whether to precast a component. Such adjustments become more manageable with the geometry schema integrated in the design concept.

It is important to note that whilst the iteration in Fig. 14 can generate various KNA components, the transverse beams along the main street were adopted from another Utzon design, prior to KNA, i.e., Schauspielhaus Zürich, a theater design project that was cancelled in 1972, due to economic inflation (Weston 2002). This Zurich beam was Utzon's adaptation of the SOH concourse beam, from his collaboration with Arup, where a U profile at span-end morphed into a T profile in midspan to support positive moment and increased the beam's rigidity, thus eliminating the need for central supporting columns (Arup and Zunz 1973). In Zurich, Utzon substituted the U and T profiles with semi-circular arcs, $\cup$ and $\cap$, and vertical panels to close the gap between the arcs. This created a folded cylindrical vault with simpler formwork for prefabrication (Nayman 2008). Although the span in the KNA beam is much shorter than that in the SOH concourse, this substitution demonstrates Utzon's ability to embed a

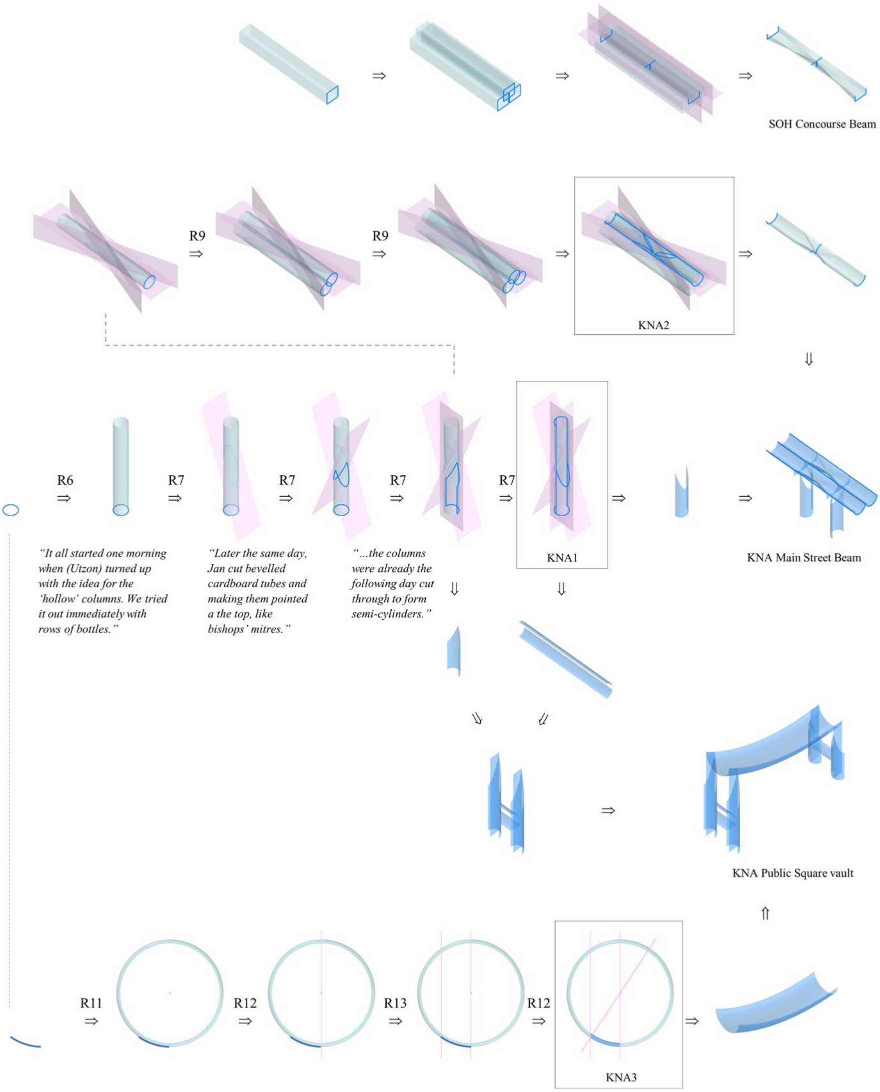

Fig. 14 Rewriting cylindrical schemas development process for the KNA components (KNA1–3). The quotes accompanying R7 iteration is from Johan Fogh's notes on KNA early experiment (Fogh 2008). An iteration for SOH concourse beam is included at the top as a reference for the main street beam design. Adapted from (Arup and Zunz 1973; Nayman 2008; Nissen 2008)

different shape while maintaining the same structural principle (i.e., folded-plate structure). This differs from the SOH roofs, where the structural principle for the same shape was changed from a shell to a rib vault system.

As can be seen in the KNA as built, this schema works to support the architect's design concept. The final cylindrical composition in KNA portrays hanging tents that shelter the parliament's activities beneath. This represents what Utzon considered to be the "Arabs' way of life and customs" (Utzon 2008, p. 5). Figure 15 shows the relationship between this reference and the final design, where the cylinders are embedded into a referenced tent and elaborated into a final design using KNA schemas. The beams imported from the Zurich theater resemble a hanging tent typically found in a traditional open-air market. The suspended concrete tent depicted above the public square and the assembly hall is derived from a toroidal schema, generated using R11-R13 and post-tensioned between the tall and short cylindrical columns as a row of box-girders (Freyssinet Report 2008). The introduction of this toroidal schema can be considered another indication of Utzon's capacity to shift different shapes across dimensions.

The Vault Schema in Farum Town Centre

Learning from the previous studies, this section further investigates Utzon's schema for his vault in the FTC design, an unbuilt project from late 1966 (Andersen 2013; Weston 2002). The main FTC design feature is the repetitive roof vaults along its shopping mall corridor, which face each other and curve down to shelter the corridor space (Fig. 16). The taller vaults are open to the south, presumably to capture and reflect the southern sunlight towards the corridor inside. The roof module consists of four components: a middle vault coupled with two side vaults along the corridor and a barrel vault that shelters the storage space. The middle vault appears to rest on an arched beam, whereas the side vaults are attached to curved cantilever columns. The roof module is repeated approximately 127 times throughout the mall complex, with some

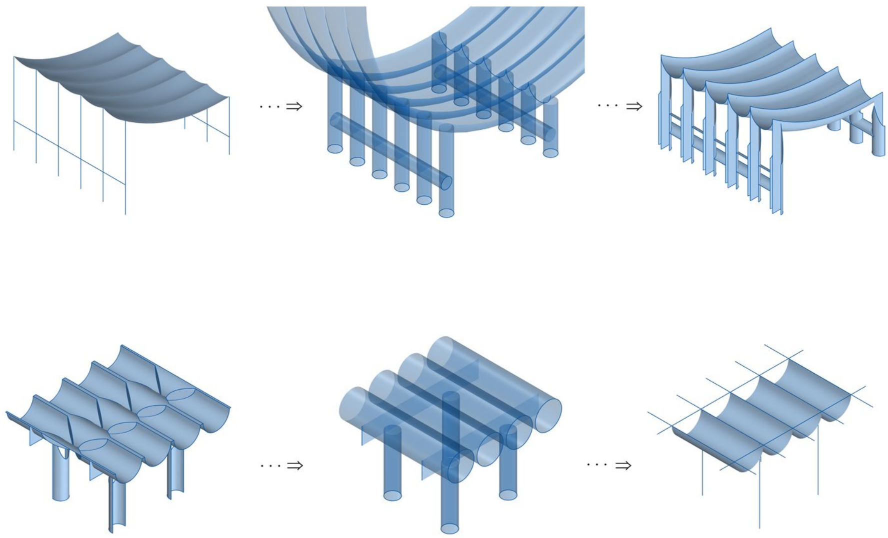

Fig. 15 Interpretation of referenced shapes and their adaptation in the KNA design via cylindrical schemas

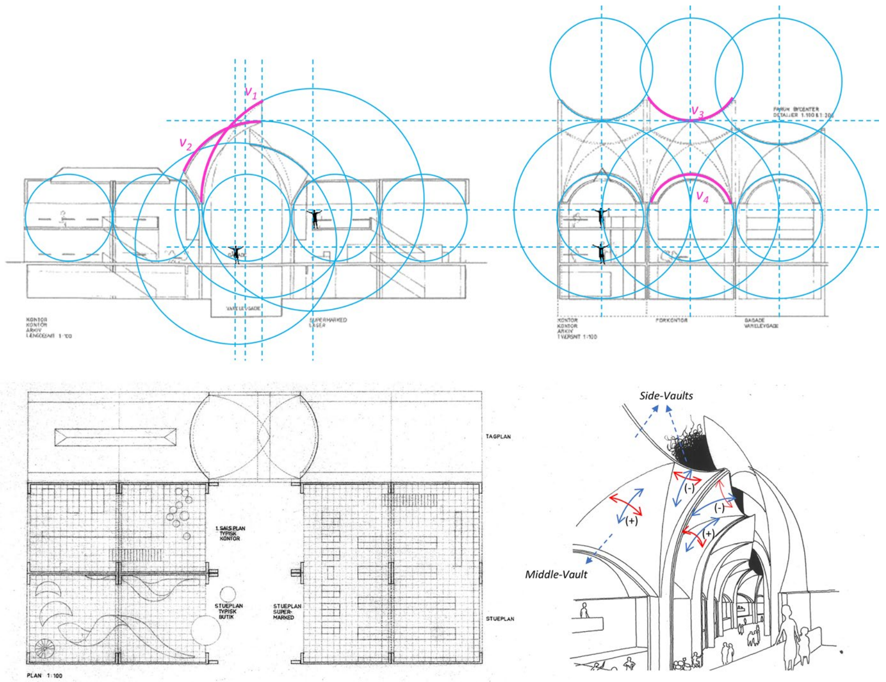

Fig. 16 The FTC competition drawings (Utzon 1966) overlaid with circles aligned with the building grid section and elevation (top). The middle-vault curves (v2 and v4) have a positive curvature, while the side-vaults (v1 and v3) have negative curvature. © Utzon Archives/Aalborg University and Utzon Center

degree of variation from straight to curved corridors and open plazas. Such a number and variety indicate a demand for an efficient repeatable module, and the consistent geometric discipline exhibited in the drawings suggests that Utzon might have formulated, or at least envisioned, a schema for an efficient structure and construction process.

As can be seen from the sections of the corridor façades, the circles confined within the main rectilinear grid are aligned with the corridor façade (Fig. 16). These circles seem to have the same radius and center points as the arch beams, and shares the same center point as the barrel-vault profile. The use of a circular grid ratio is also visible; for instance, the middle vault radius in the tall roof is twice the width of one of the stores in the mall. More interestingly, the vault profile is not only aligned with the circles, but their center points seem to coincide with the standard height of a person on the ground floor and at the mezzanine level. The alignment to human scale indicates that the structure's form and function might have been elaborated during the design process, instead of as an afterthought as in the case of the SOH. This reading makes the roof vault design appear enigmatic and intriguing. Although a curved surface and circular shape were used extensively in the FTC, it is not clear whether they were derived from a sphere. The middle vault resembles the SOH's sails with a positive curvature, which might be derived from a sphere or a pointed dome. However, the side vaults indicate a negative curvature, recalling the inner side of a torus as in KNA vault. There is no documentation of a geometric schema for these vaults.

To deduce schema of Farum's vault, the first interpretation uses a sphere, whereas the second and the third interpretations use a torus and a pointed dome. To highlight their differences, the middle vault (magenta) is set as a parametric shape, while the side vaults (cyan) are fixed by the store-width and projected as a torus's negative curvature segment. The arcs and circles (as the arcs' inversed part) in Fig. 17 serve as the initial condition, from which R1 creates a torus segment T^- (the superscripts − and + indicate respectively the segments of negative and positive curvature of a torus). R3 slices T^- with plane P_A resulting in arc *a-b-c* and R2 slices T^- with plane P_B, creating arc *a′-b′-c′* (with *c″* is defined as the middle point of line |a′-b′|). In the next step, the iteration is split into three paths to demonstrate different possible shapes for the middle vault.

Deviating to the iteration on the right in Fig. 17, circle C_2 is redrawn with R1 and R3 slices C_2 with plane P_C. R1 then inverses C_2 into a sphere *Sp* such that T^- and *Sp* intersect with each other, and also with P_A and P_C. R5 then captures the side vaults from T^- and the middle vault from *Sp*. The results and parameters from this step are formulated into Farum Schema 1, where o_2 can move along the store's center axis to change the middle-vault curvature, and P_C can rotate with angle α to change its bottom arc opening.

The second iteration on the left considers the middle vault and side vault segments as parts of the same torus *T*. R1 inverses C_1 and v_4 (as a mirrored version of v_3) to create a torus segment with positive curvature T^+ (magenta). R4 then rotates T^+ at *a′* so that its revolving axis coincides with *c*. In this rotation, P_C is also copied from P_B. From the two-overlapping toruses, R3 intersects T^+ and T^- with each other, and also with P_A and P_C to divide them into segments, and R5 captures the middle and side vaults from the torus. This step is formulated into Farum Schema 2. As the modules between the positive and negative segments along the sectional ring of a torus ring would always be different regardless, this schema allows the T^+ and T^- profiles to be adjustable while sharing the same center point and radius of revolution.

The vaults that result from Farum schemas 1 and 2 resemble Utzon's FTC vault, yet with a rounded corner instead of a sharp corner at the tip of the middle vault. The middle iteration in Fig. 17 solves this problem by using a pointed dome. R13 adds plane P_C, perpendicular to line |*c–c″*| and R11 redraws C_4, coplanar with P_C with a radius equal to |c″-a′|. A new rule R22 creates a pointed dome *D* by revolving v_2 along C_4, such that *D* and T^- intersects with each other and with P_A and P_C. R5 captures a middle vault from *D* and the side vaults from T^-. This step is formulated into Farum Schema 3. With *D* constrained to T^- parameters, the only inputs required are the ones that make T^- (i.e., C_1 and v_3).

Using the rules from the previous projects and a new three-dimensional shape that has a circle as it boundary (the pointed dome), it was possible to generate quick interpretation of the Farum schemas (Fig. 18). While schemas 1 and 2 are simple and flexible and generate shapes similar to the FTC design, the resulting shape appears slightly different from the intended design. In contrast, Schema 3 produces a

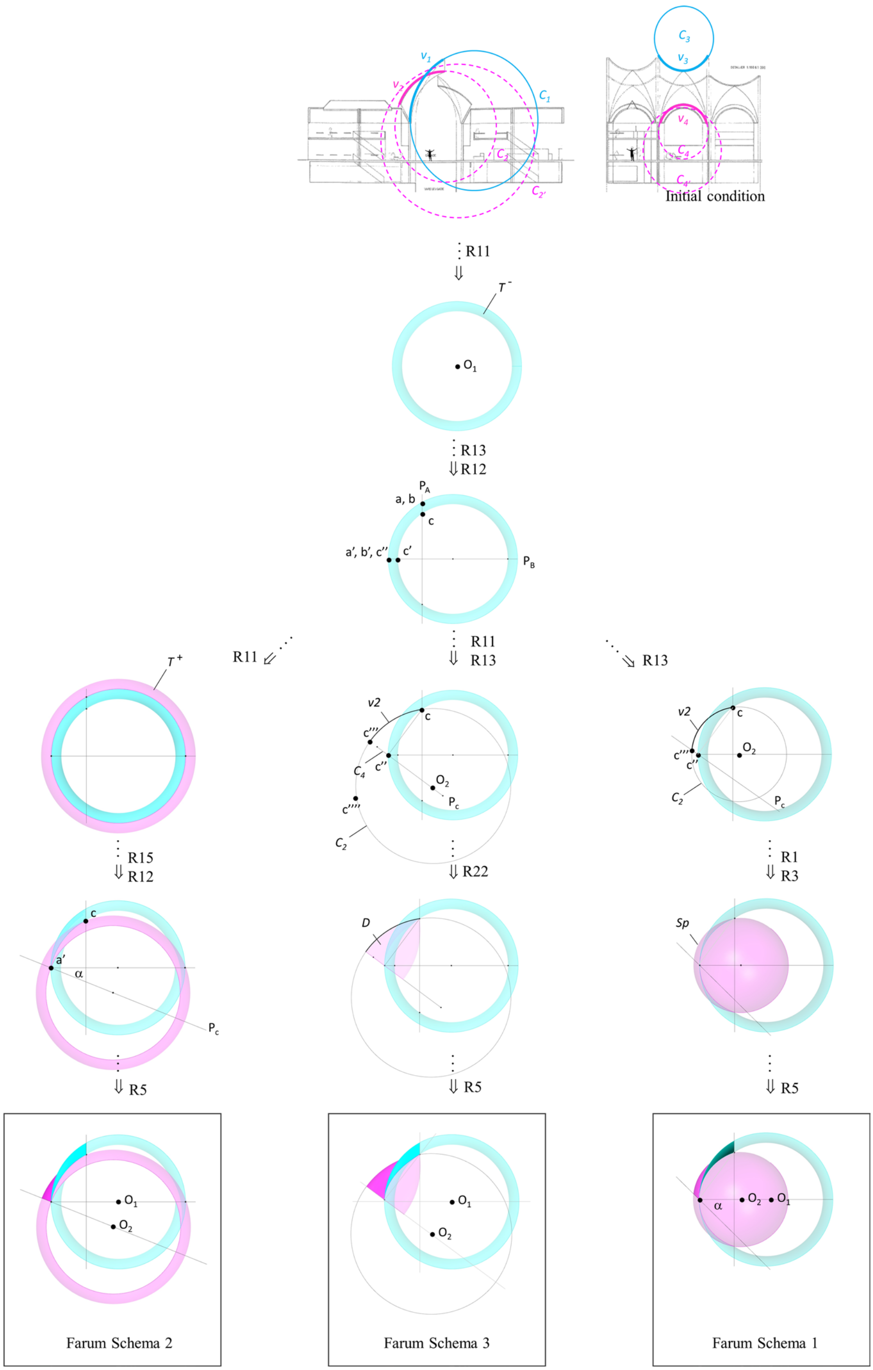

Fig. 17 Interpretation of the Farum vault schemas. The side vaults (cyan) are fixed as a part of torus while the middle vault (magenta) is adjustable and its curves' inverse-boundary is open for interpretation. Farum schema 1 uses a sphere (right), schema 2 uses torus (left) and schema 3 uses a pointed dome (middle)

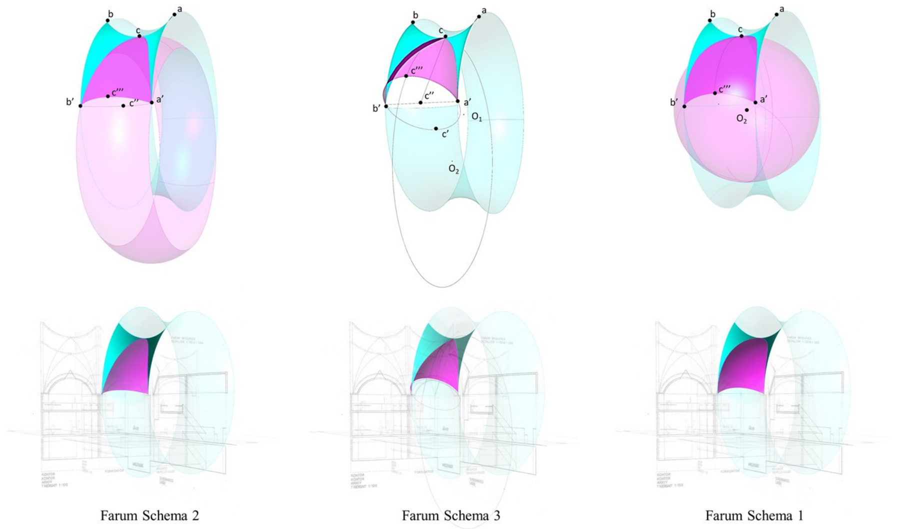

Fig. 18 Continued from Fig. 17, showing the schemas from behind (top) and the front (bottom). Schema 3 produces a vault that resembles the original design better than the others

shape that matches the design more closely with a sharp corner, yet it is less flexible due to tightly constrained shapes and a smaller number of parameters. Nevertheless, all three schemas use basic shapes that can be tessellated into standardized panels if needed.

Conclusion

This paper has outlined a series of interpretative studies on Utzon's ideation process in producing vault schemas, from recapturing the eureka moment of the spherical schema, to interpreting the schemas for the SOH ribs, tiles, and acoustic ceiling; and from rewriting the BC and KNA schemas, to reimagining different schemas for the FTC vaults. The shape grammar analysis shows that interpreting the schema ideation process is productive for understanding not only how an architect develops schema to solve various problems within a project but also how the underlying principles can work for other projects. Cross-dimensional embedding contributes greatly to the ideation process. While initially starting with a sphere, the investigation presented here showed other various shapes that have a circular curve as their boundary (e.g., from a sphere to a cylinder and from the torus to the dome). Although some of these shapes and transformation methods recur across different schemas (the cylinder, for instance, reappeared in the SOH Minor Hall, BC, and KNA, and the shape-subdivision was employed in SOH and KNA), they evolved from different design concepts and aided various functions, such as the use of a cylinder to mimic water reflecting sunlight to optimize a room's acoustics, or recreating a cloud's atmospheric ambiance to augment interior daylight.

The single-curvature shape with a standard size that was used in several schemas provided a simpler formwork and minimized the number of unique parts. This can be seen in the KNA and the SOH Minor Hall schemas, which used cylinders with a constant diameter. It is also visible in the BC schema, which used cylindrical segments, extruded in one direction, so that they could be cast with repetitive formworks. The use of a double-curvature shape such as that for the SOH ribs and panels of tiles, the KNA folded beam, and the FTC vaults, is justified by the frequency of its repetition.

This study also highlights different transformation strategies in the schemas' development. These include: subdivision (as applied in the SOH sphere, the KNA cylinder, and the FTC vaults); substitution (such as the replacement of the SOH concourse beam profiles in the KNA beam and the exchange of shapes in generating several Farum schemas); and addition (as illustrated in the Minor Hall ceiling schema generation). Some schemas, such as Minor Hall and BC, share the same basic shape and resemble each other, yet serve different purposes. Others use different shapes that were developed for the same purpose. For example, both the SOH spherical schema and KNA cylindrical schema subdivide a shape to generate various prefabricated components.

It is not within the scope of this paper to investigate the structural and acoustic performance of each of Utzon's vaults in detail. As the vault designs in this exercise are approximations, future studies will be necessary to assess the schemas' capacity to handle technical constraints more accurately. Nevertheless, explicit parameters in schemas can transform vault designs into more straightforward designs for the purpose of further simulation and optimization. By using basic geometry composed of identical curvatures (e.g., sphere, dome, torus, or cone), the vault also becomes easier to tessellate and more manageable to build.

Whilst the schema in this study is not a substitution for the intuitive design process, the study on the four projects suggest a different level of integration between the schema development and design process. In the case of the SOH, the spherical schema came after the design was finished. In BC and KNA, the cylindrical schemas evolved along with the design process, which also seems to be the case in the FTC, as indicated by the geometric alignment of the building components and human scale. Utzon's use of domestic objects during his ideation stages, such as sugar cubes, a beach ball, and a bathtub, also suggests that he managed to distance himself from the formality of the schema. He seems to have been to be fully aware that there was a geometric mechanism ready to translate his intuition into an explicit representation of it for construction and communication purposes. This awareness managed to bridge the gap between intuition and computation in his projects, as demonstrated in his rigorous, yet versatile geometric schema to rationalize the intuitively driven ideas.

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References

Andersen, Michael Asgaard. 2013. *Jorn Utzon: Drawings and Buildings*. New York: Princeton Architectural Press.

Arup, Ove and Jack Zunz. 1973. Sydney Opera House 1973. *The Arup Journal*: 4–21.

Fogh, Johan. 2008. The Genesis of the Kuwait Project. In *Jorn Utzon Logbook: v. 4: Kuwait National Assembly*. Hellerup: Edition Blondal.

Freyssinet Report. 2008. General Meeting of Agents - June 92. In *Jorn Utzon Logbook: v. 4: Kuwait National Assembly*. Hellerup: Edition Blondal.

Jensen, Godtfred. 2005. Building System. In *Bagsvaerd Church: Jorn Utzon Logbook: v. 2*, ed. Torsten Blondal, tran. W. Glyn Jones. Hellerup: Edition Blondal, Denmark.

Leplastrier, Richard. 2017. Utzon Legacy presented at the BDES3027 Guest Lecture, September, The University of Sydney.

Mikami, Yuzo. 2001. *Utzon's Sphere Sydney Opera House - How It Was Designed and Built*. 1st edition. Tokyo: Shokokusha Tokyo.

Mortensen, Bo. 2005. The Acoustic in Bagsvaerd Church. In *Bagsvaerd Church: Jorn Utzon Logbook: v. 2*, ed. Torsten Blondal, tran. W. Glyn Jones, 152–154. Hellerup: Edition Blondal,Denmark.

Nayman, Oktay. 2008. Folded Beams (Letter to Richard Weston - 2006). In *Jorn Utzon Logbook: v. 4: Kuwait National Assembly*. Hellerup: Edition Blondal.

Nissen, Borge. 2008. Geometry. In *Jorn Utzon Logbook: v. 4: Kuwait National Assembly*, 122–126. Hellerup: Edition Blondal.

Stiny, George. 1993. Boolean algebras for shapes and individuals. *Environment and Planning B: Planning and Design* 20: 359 – 362. https://doi.org/10.1068/b200359.

Stiny, George. 2011. What Rule(s) Should I Use? *Nexus Network Journal* 13: 15–47. https://doi.org/10.1007/s00004-011-0056-6.

The Sydney Opera House Trust. 2020. Spherical Solution. *Sydney Opera House*. https://www.sydneyoperahouse.com/content/soh/our-story/sydney-opera-house-history/spherical-solution.html. Accessed July 9.

Utzon, Jorn. 1962a. *The Yellow Book*. The State Records Authority of New South Wales.

Utzon, Jorn. 1962b. SOH 640 - Geometrical construction showing the shells of major hall. © Utzon Archives/Aalborg University & Utzon Center.

Utzon, Jorn. 1965. Sydney Opera House: The Roof Tiles. *Architecture in Australia*, December.

Utzon, Jorn. 1966. FARUM BYCENTER_L_D030_002. © Utzon Archives/Aalborg University & Utzon Center.

Utzon, Jorn. 2008. *Jorn Utzon Logbook: v. 4: Kuwait National Assembly*. Hellerup: Edition Blondal.

Utzon, Jorn, Jensen Godtfred, Bo Mortensen, Martin Schwartz, Svend Simonsen, Richard Weston, Jan Ulzon, and Lin Ulzon. 2005. *Bagsvaerd Church: Jorn Utzon Logbook: v. 2*. Ed. Torsten Blondal, trans. W. Glyn Jones. Hellerup: Edition Blondal,Denmark.

Weston, Richard. 2002. *Utzon: Inspiration, Vision, Architecture*. Edition Blondal.

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